GEOMETRY
CORE LEARNING OUTCOMES

Slovak Society for Geometry and Graphics

Visegrad Fund

International Visegrad Fund

Small Grant 11520208

GEOMETRY
CORE LEARNING OUTCOMES

Slovak Society for Geometry and Graphics

The strategic document Geometry Core Learning Outcomes was created under the International Visegrad Fund project
International Symposium on Geometry and Graphics
Small Grant ID 11520208.

Visegrad Fund

Foreword

The V4 countries bound by common space of territory, culture and especially last decades' history face similar problems in education and research. During this period their education systems have been undergoing the changes in law, content and scope at all levels and types of schools. Considering mathematics, the applied changes resulted in dramatic reduction of number of teaching lessons and curriculum liberalization, weakening the general consideration of mathematics importance, leading to a dramatic decline in knowledge and skills of students. At tertiary education, the alarming situation has been widely recorded in all Central European countries, when the applicants come to study with very poor readiness as well as the low interest to study technical and natural sciences programs.

The document brings the glance into education of Geometry, one part of mathematics, in each from V4 counties: Slovakia, Czech Republic, Poland, and Hungary, where authors representing National Societies for Geometry and Graphics inform on evolution of education in their countries during previous 10-20 years and point out on importance of geometry competencies in everyday life and for future engineers. Moreover, teachers and other interested parties can find here overall lists of topics related to geometry, arranged in transparent summary tables, taught at secondary schools as well as at universities with special attention given to technical faculties. This summary provides an important source in specification of curriculum and teaching methodology for building necessary background skills and levels of understandings fundamental characteristic properties of basic geometric concepts leading to steady applicable knowledge of learners.

Daniela Richtáriková

Geometry Education at Slovak Universities of Technology

Daniela Velichová

Slovak Society for Geometry and Graphics
KNám. Slobody 17, 812 31 Bratislava, Slovakia
email: daniela.velichova@stuba.sk

The transition of the higher education system in Slovakia in years 2002 - 2004 to the 3-level structure (when bachelor, master and PhD. study programmes were introduced) considerably influenced the extent and contents of basic courses at the first level programmes at the technical universities. Natural sciences, including physics, mathematics and geometry as a separate subject, were the most reduced once. The next reform of the primary and secondary school system in the country according to the Law No. 245/2008 of the Codex of laws on education passed by the Slovak government on the May 22, 2008 that started to be realised from September 1, 2008 incredibly diminished amount of lessons for mathematics and physics at the primary and secondary education. Mathematics was no more a subject at the secondary school leaving examination - maturita, and there was no pressure on choosing these optional subjects for the final exam as a necessary precondition to apply for the technical university study programmes. These circumstances change the general knowledge of secondary school graduates who became university freshmen into such an extent that it was necessary to adjust the contents of the basic courses at the bachelor level of study. This was the crucial point for the Slovak tertiary education system that had to be adapted to the expected lower level of the secondary school graduates. In connection to the following reforms passed by the Law No. 455/2012 on the Higher education from September 2012, the ECTS credit system was accepted in co-ordinance with the European standards and the accreditation criteria were passed. Last general accreditation of all university study programmes has been realized in the years 2014-2015, with the results establishing new study programmes at all higher education institutions in Slovakia from the academic year 2015/16. In these newly accredited programmes, more attention is drawn to the general knowledge of university graduates related to their professional orientation than to the transition problems of the first year students at the bachelor study programmes. Existed lack of steady background in mathematics and other natural sciences is not reflected in the composition of bachelor study programmes, and what is even worse, no specific obligatory subjects on higher level advanced mathematics are included in the master and PhD. study programmes, just a few once offered as optional subjects. These will lead to a further decline in knowledge not just in the natural sciences themselves, but consequently to the overall level of scientific development in applied sciences and technical disciplines, which are inherently dependent on the advances in applied mathematics and advanced mathematical methods and theoretical results.

Geometric thinking and reasoning is the most natural way of applying mathematical methods in technical disciplines. Geometric intuition is a basic intellectual potential of technicians, engineers who realise theoretical models described in terms of abstract physical rules and mathematical formulas into real physical objects. To present the learning outcomes of the current university graduates from geometry, after passing all 3 levels of national education system in Slovakia, the following tables are inserted, focused on the scope of the possible acquired basic geometric knowledge and skills.

Status of topics related to geometry included in the curricula for the secondary schools in Slovakia is presented in the Table 1.

Table 2. brings an overview of geometry subjects still taught at the various faculties of technical universities in Slovakia, while Table 3. presents the most frequent geometric topics appearing in the obligatory subjects, including basic courses on Mathematics I and II at the technical universities in Slovakia.

The sad fact that has to be mentioned here is that there is low level of interest among young people in Slovakia to study natural science at all, not mentioning the almost empty set of those interested in study of traditional pedagogical combinations of Mathematics and another subject as Biology, Descriptive geometry, Chemistry, Geography, Physics, apart from Informatics. The few graduates, in addition, do not usually enter the profession of school teachers, as they prefer much better paid positions in the information technology fields. The average age of secondary school teachers in the year 2014 was about 45 years, while the majority of teachers that are pensioners are teachers of biology, chemistry, geography, or mathematics and physics. Prognosis for the coming next 10 years is not very optimistic, as there can be expected a crucial problem to appear in the supply of well educated pedagogical staff for natural sciences at the secondary schools in Slovakia. This situation can be regarded as an unpleasant consequence of the neglecting the overall development, the decline and lack of interest in natural sciences in the young generation of pupils from the primary up to the secondary schools, not excluding university students.

Table 3 shows the range of the geometric knowledge that students should have after their university studies at the technical faculties. The depth and level of the acquired knowledge objectively depends very much on the specific professional orientation of engineering students. It is considerably more profound at the architecture, civil engineering and mechanical engineering programmes, where sole subject on geometric methods, basic properties of spatial geometric objects (surfaces and solids), and projection methods of the space to the plane retained as essential information and topics in differently renamed subjects substituting traditional courses on Descriptive geometry. Here the spatial understanding and 3D geometry remained as a necessary part of the professional profile of the study graduates.

Table 1. Topics on Geometry covered by curriculum of Mathematics at the secondary schools in Slovakia according to the Regulation of the Ministry of Education of the Slovak republic from 2005.

Secondary schools (young people aged 15 - 18)	
Basic range – vocational schools, apprenticeship (maturita), art schools 28 hours / year	Extended range (basic range and moreover) – grammar schools 40 hours / year
Planimetry	
1. Geometry of triangles – theorems of Pythagoras and Euclid, similarities and equality, various constructions based on inscribed and superscribed circles and calculations of measures using goniometric functions (area, angles, length, circumference).	1. Constructions of triangles bases on special sets of points of given properties.
2. Geometry of circle - central angle and inscribed angle, tangent to the circle, theorem of Thales, properties of tangent circles, circumference and area.	2. Quadrangles inscribed and superscribed to a circle, squares described on a circle, common tangents to 2 circles.
3. Convex quadrilaterals – sum of interior angles, constructions, area and circumference	3. Convex *n*-gons – number of diagonals, sum of interior angles, some constructions (pentagon, hexagon), area, circumference of a planar region. 4. Planar mappings (isometries and homothety) – constructions of planar figures using geometric transformations. 5. Definition and basic properties and constructions of ellipse, hyperbola and parabola

Cartesian geometry in the plane	
1. Vectors and operations on vectors in plane and in space.	1. Linear combinations of vectors, colinearity and complanarity of points.
2. Equations of a straight line and a plane (general, parametric), superposition of basic linear figures in the space.	2. Calculation of intersection points and lines of linear space figures.
3. Distance of 2 points, 2 planes, point and plane; angle of 2 lines, 2 planes and line and plane.	3. Distances and angles of basic linear figures in the plane.
4. Equations of circles, set of common points of a line and a circle, equation of tangent to a circle at the point.	4. Equations of conic sections – ellipse, hyperbola, parabola, basic properties .
Stereometry	
1. Basic space elements and their relations – 2 lines, line and plane, 2 – 3 planes, angles of lines and planes on cube, and their views in parallel projection.	1. Calculations of distances and angles of lines and planes in space, understanding of basic properties of parallel projection.
2. Basic elements of prisms (cube, and truncated) and pyramids (including truncated and tetrahedron), calculations of volume and surface area, length of edges, altitudes, angles between edges, face areas.	2. Calculations of metric problems on prisms and pyramids (including truncated), basic properties s of other regular polyhedral..
3. Basic elements of cylinders, cones and sphere (including truncated), calculations of volume and surface area, angle between axis and lines, length of altitude.	3. Calculations of metric problems on truncated cones and parts of spheres.

Table 2. Geometric subjects at various faculties at technical universities in Slovakia

Faculty	Subject – Extent Lectures/Practicals
Civil Engineering	
1. STU in Bratislava	Descriptive geometry I – 2/2 Descriptive geometry II – 2/2
2. University in Žilina	Descriptive geometry – 2/2
3. TU Košice	Descriptive geometry I – 2/2
Architecture	
1. STU in Bratislava	Descriptive geometry I – 2/2 Descriptive geometry II – 2/2
Design (Art)	
1. TU Košice	Descriptive geometry I – 2/2
Mechanical Engineering	
1. STU in Bratislava	Constructive geometry – 2/2
2. University in Žilina	Constructive geometry – 2/2

Table 3. Geometric topics at the technical universities in Slovakia

Bachelor study programmes (young people aged 18/19 - 21/22)
Mathematics I at all TU 1st semester, 24 hours / semester
1. Basics of vector calculus in 3-dimensional Euclidean space, equations of planes and lines, calculations of intersections and their position, metric properties. 2. Sketching graphs of functions, tangent lines at points and their asymptotes (with equations), extremes, convex and concave behaviour, points of inflection.

3. Definite integrals and their geometric applications - sketching and calculations of surface area of planar regions, length of a curve segment, volume and surface area of surfaces of revolution.

Mathematics II at all TU
2^{nd} semester, 24 hours / semester

1. Equations of quadric surfaces, calculation of their intersections.
2. Sketching graph of functions in 2 variables, point on graph and tangent lines to iso-parametric curves, normal to tangent plane, geometric representation of constrained extremes of functions of 2 variables.
3. Geometric representation of constrained extremes of functions of two variables.
4. Multiple integrals and their geometric applications – sketching and calculations on solids bounded by quadric surfaces or planes (solid volume and coordinates of its centre of gravity), space curve length, surface patch area.

Applied Mathematics at all TU
2^{nd} semester, 24 hours / semester

1. Differential geometry of curves – definition, vector equation, differential properties, first and second curvatures, Frenet-Serret moving trihedron, calculation of curve segment length, views of curve segments in orthogonal axonometry.
2. Differential geometry of surfaces – definition, vector equation, differential properties, tangent plane and normal, types of points on surfaces, first and second fundamental forms, mean and Gaussian curvatures, calculation of surface patch area, classification of surfaces, views of surface patches.
3. Applications of curve and surface integrals, calculation of surface patch area and solid cell volume.

Descriptive Geometry I, Civil Engineering Faculty STU Bratislava
1^{st} semester, 52 hours / semester

1. Basics of parallel projection, orthographic projections.
2. Conic sections – properties, equations, constructions.
3. Axial affinity and central collineation, basic properties, matrix representations, constructions.

4. Monge method MP, and quoted projection – views of points, lines and planes and their position.
5. Basic geometry of curves and surfaces – definitions, equations, tangent line and normal, tangent plane, osculating plane, curves on surfaces, graphical curves and surfaces.
6. Topographical surfaces and their applications.
7. Axonometric projections – basic properties and constructions, Polkhe theorem, block diagram of topographical surfaces, views of basic geometric figures in orthogonal axonometry OA.
8. Surfaces of revolution – definition, properties (meridians, special parallel circles), tangent and normal cones, views in MP and OA.
9. Cartographic nets on sphere and ellipsoid of revolution in OA.
10. Construction of surfaces of revolution in CAD systems.

Descriptive Geometry II, Civil Engineering Faculty STU Bratislava
2nd semester, 52 hours / semester

1. Basics of central projection, metric problems.
2. Basics of linear perspective – properties, block-diagram in linear perspective, 1-3 vanishing point perspective.
3. Stereoscopic projections – properties, equations, views of a point, principles of stereoscopy.
4. Basics of photogrammetry – basic relations and equations, projective properties of a photographic image, calibration of images, reconstruction of objects from various images.
5. Cylindrical and conical perspective – definitions, properties, views of figures, development.
6. Geometric backgrounds of cartography – various reference surfaces of the Earth, cartographic projections and their classification, views of cartographic net in orthographic, stereographic and gnomonic projections, cylindrical and conical projections, views of orthodromes and loxodromes.

Descriptive Geometry, Civil Engineering Faculty Univerzity of Žilina
1st semester, 52 hours / semester

1. Basics of parallel projection, orthographic projections.
2. Conic sections – properties, equations, constructions.
3. Axial affinity and central collineation, basic properties, and constructions.
4. Monge method MP, and quoted projection – views of points, lines and planes and their position.

5. Axonometric projections – basic properties and constructions, Polkhe theorem, block diagram of topographical surfaces, views of basic geometric figures in orthogonal axonometry OA.
6. Basic geometry of curves and surfaces – definitions, equations, tangent line and normal, tangent plane, osculating plane, curves on surfaces, graphical curves and surfaces.
7. Elementary surfaces and their planar intersections in all projection methods.
8. Topographical surfaces, level curves, slope of a surface, intersections of topographical surfaces, profile and cylindrical profile.
9. Block-diagrams, views of topographical surfaces in axonometry.
10. Surfaces of revolution, cartographic net on sphere in OA.
11. Surfaces of constant slope and their connection to terrain.

Descriptive Geometry, Civil Engineering Faculty TU Košice
1st semester, 52 hours / semester

1. Basics of extended Euclidean space, axial affinity and central collineation, basic properties.
2. Conic sections – properties, constructions, tangent to curve.
3. Basics of Monge method MP – views of points, lines and planes.
4. Elementary solids, views in MP – intersections with planes and lines (prism, pyramid, cylinder, cone, sphere).
5. Basic of orthogonal axonometry – definition, metric properties, views of points, lines and planes.
6. Elementary solids, views in OA – intersections with planes and lines (prism, pyramid, cylinder, cone).
7. Basics of central projection – definition, metric properties, views of points, lines, planes and planar figures, planar intersections of solids.
8. Linear perspective – definition and basic properties, view opf a circle, views of elementary solids.
9. Surfaces of revolution – definition, properties (meridians, special parallel circles), tangent and normal cones, planar intersections, special quadric surfaces (ellipsoids, hyperboloids, paraboloids).
10. Ruled surfaces – definition, properties, classifications.
11. Helicoids and helical surfaces – definition, properties (meridians, special helices), tangent and normal cones, intersections by meridian and normal planes.

Descriptive Geometry, Faculty of Arts TU Košice
1st semester, 52 hours / semester

1. Basics of extended Euclidean space, axial affinity and central collineation, basic properties.

2. Conic sections – properties, constructions, tangent to curve.
3. Basics of Monge method MP – views of points, lines and planes.
4. Elementary solids, views in MP – intersections with planes and lines (prism, pyramid, cylinder, cone, sphere).
5. Basic of orthogonal axonometry – definition, metric properties, views of points, lines and planes.
6. Elementary solids, views in OA – intersections with planes and lines (prism, pyramid, cylinder, cone).
7. Basics of central projection – definition, metric properties, views of points, lines, planes and planar figures, planar intersections of solids.
8. Linear perspective – definition and basic properties, view opf a circle, views of elementary solids.
9. Surfaces of revolution – definition, properties (meridians, special parallel circles), tangent and normal cones, planar intersections, special quadric surfaces (ellipsoids, hyperboloids, paraboloids).
10. Ruled surfaces – definition, properties, classifications.
11. Helicoids and helical surfaces – definition, properties (meridians, special helices), tangent and normal cones, intersections by meridian and normal planes.

Constructive Geometry, Mechanical Eng. Faculty STU Bratislava
3rd semester, 52 hours / semester

1. Basics of extended Euclidean space, homogeneous coordinates.
2. Conic sections – properties, equations, constructions.
3. Axial affinity and central collineation, basic properties, matrix representations, constructions.
4. Basics of projections of space – parallel (orthographic), central.
5. Orthographic projection methods: Monge method MP, and orthogonal axonometry OA – views of points, lines and planes.
6. Elementary surfaces and solids, views in MP and OA, intersections with planes and lines (prism, pyramid, cylinder, cone, sphere).
7. Basic geometry of curves – definition, differential properties, curvatures, Frenet-Serret trihedron, rectifications.
8. Special curves of technical praxis (helix, approximation and interpolation curves), generating principles, equations, properties, views in MP and OA.
9. Basic geometry of surfaces – definition, differential properties, tangent plane and normal, planar intersections, intersections of line and surface, intersections of surfaces, classification of surfaces.
10. Developable surfaces – definition, Catalan theorem, development of surface patches.
11. Surfaces of revolution – definition, properties (meridians, special parallel circles), tangent and normal cones, planar intersections,

special quadric surfaces (ellipsoids, hyperboloids, paraboloids). 12. Helicoids and helical surfaces – definition, properties (meridians, special helices), tangent and normal cones, intersections by meridian and normal planes. 13. Envelope surfaces – definition, properties, characteristics, Archimedean serpentine.
Constructive Geometry, Mechanical Eng. Faculty University in Žilina 1st semester, 52 hours / semester
1. Basics of extended Euclidean space, homogeneous coordinates. 2. Conic sections – properties, equations, constructions. 3. Axial affinity and central collineation, basic properties, matrix representations, constructions. 4. Basics of projections of space – parallel (orthographic), central. 5. Orthographic projection methods: Monge method MP, and orthogonal axonometry OA – views of points, lines and planes. 6. Elementary surfaces and solids, views in MP and OA, intersections with planes and lines (prism, pyramid, cylinder, cone, sphere). 7. Basic geometry of curves – definition, basic properties. 8. Special curves of technical praxis – helix, properties, equation, views in MP and OA. 9. Basic geometry of surfaces – definition, tangent plane and normal, planar intersections, intersections of line and surface, intersections of surfaces, classification of surfaces. 10. Developable surfaces – definition, Catalan theorem, development of surface patches. 11. Surfaces of revolution – definition, properties (meridians, special parallel circles), tangent and normal cones, planar intersections, special quadric surfaces (ellipsoids, hyperboloids, paraboloids). 12. Helicoids and helical surfaces – definition, properties (meridians, special helices), tangent and normal cones, intersections by meridian and normal planes.

One of the most important issues in order to reverse this undesirable situation is to establish priorities for the level of mathematical education at all types of schools, and to formulate real need of graduated teachers of mathematics and curriculums for Mathematics and Descriptive Geometry as a continuation of general education at the level of higher education.

Geometry Education at Universities of Technology in the Czech Republic

Světlana Tomiczková

Czech Society for Geometry and Graphics
Technická 8, 301 00 Plzeň, Czech Republic
Email: svetlana@kma.zcu.cz

Czech higher education dates back six hundred years. In 1348 Emperor Charles IV founded a university in Prague which is the oldest academic institution in the Central Europe, now called the Charles University. The oldest technical university in the Czech Republic is the Czech Technical University in Prague (Bohemian Polytechnica) founded in 1707. Nowadays, there exist 26 public universities, 2 state (police and military) universities and many further private institutions of higher education in the Czech Republic. The central governing body for education is the Ministry of Education, Youth and Sports. The quality of higher education is fostered by the Accreditation Commission.

Since 2001 the three cycle structure has strictly been implemented in the higher education system (i.e., Bachelor's, Master's and Doctoral study programmes). In addition, the ECTS credit system started to be introduced to the Czech higher education that time. Nowadays, it is widely used throughout the higher education institutions in the Czech Republic as it facilitates student mobility within Europe and the comparison of study programs and courses. It must be stressed out that the mentioned reforms significantly influenced the extent, contents and subject syllabi of all fundamental courses mainly at the first level study programmes at all universities, including the technical ones. Especially natural sciences, including mathematics and geometry, were affected, in many cases substantially reduced to leave the place to other subjects of professional orientation.

In line with the new curricular policy principles outlined in the National Education Development Programme for the Czech Republic (the so-called "White Paper") and enshrined in the Education Act (on Pre-school, Basic, Secondary, Tertiary Professional and Other Education), a new curricular system for pupils and students from 3 to 19 years of age is being introduced into the Czech education system in 2004. Curricular documents are developed at two levels: the national level and the school level. This step as a by-product significantly influenced a level of mathematics and (descriptive) geometry courses at the Czech universities of technology despite it dealt originally with primary and secondary schools only. Amount of lessons for mathematics and natural sciences at the primary and secondary education incredibly diminished.

Mathematics has been no more a subject at the secondary school leaving examination (maturita) and since that there has been no pressure on choosing it for the final exam as a necessary precondition to apply for study programmes and study fields at the universities of technology. This was reflected soon in a knowledge of applicants, subsequently followed in curricula changes. Many, in the past presumed, topics must have been integrated to fundamental bachelor courses at universities, which had to get adapted to the expected lower level of the secondary school graduates.

One of the main problems is that the evident lack of a firm background in mathematics, geometry and natural sciences is still not reflected appropriately in the composition of bachelor study curricula. Moreover, in many study programmes there are no obligatory courses on advanced mathematics and geometry neither in master and PhD. study programmes. Already, this lead to a further decline in knowledge of students not just in the natural sciences themselves, but subsequently to the overall level of scientific development in applied sciences and technical disciplines.

As well-known, geometric thinking and reasoning and geometric skills are the most natural way of applying mathematical methods on challenging technical problems. Geometry is especially important for technicians, engineers, graphic designers, architects and others who realise theoretical models described in terms of abstract physical rules and mathematical formulas into real physical objects and shapes. Unfortunately, as in the other countries, there is low level of interest among young people to study mathematics, natural and technical sciences. And the same also holds for studying pedagogical combinations of Mathematics and another subject.

In the following several tables, we present the learning outcomes from geometry courses, specializing only on graduates from technical universities in the Czech Republic. Status of topics related to geometry included in the curricula for the secondary schools in the Czech Republic is presented in the Table 1. Table 2 brings an overview of geometry subjects still taught at the various faculties of technical universities in the Czech Republic, while Table 3 presents the most frequent geometric topics appearing in the obligatory subjects, including basic mathematical courses at the technical universities in the Czech Republic. Table 3 shows the range of the geometric knowledge that students should have after their university studies at the technical faculties. The depth and level of the acquired knowledge objectively depends significantly on the specific professional orientation of engineering students and on a particular university or faculty.

Table 1: Topics on Geometry covered by curriculum of Mathematics at secondary schools in the Czech Republic according to the Regulation of Ministry of Education of the Czech Republic.

Secondary grammar schools and Technical lyceum
Geometry in the plane (planimetry)
Polygons and Circles and their properties, area, perimeter. **Triangles** – similarity, congruency of triangles, theorems of Pythagoras and Euclid, construction based on special sets of points of prescribed properties. **Circles** – central and inscribed angle, tangent, theorem of Thales **Planar mappings** – isometries and similarities, constructions based on these mappings.
Geometry in the space (stereometry)
Fundamental solids – prism, pyramid, cylinder, cone, sphere and their properties – volume, area, angles. **Orthogonal projection** - basic features of elements in the space.
Analytic geometry
Vectors and operations with vectors. Equations of the **straight line** in the plane. **Conic sections** (circle, ellipse, parabola, hyperbola – equations, intersections of a conic section and a straight line, tangent line.
Trigonometry
Law of sines, law of cosines, arbitrary triangle, right-angled triangle

Table 2: Geometric subject at various faculties at technical universities in the Czech Republic.

Faculty	**Subject**
Civil Engineering	
Czech Technical University in Prague - Faculty of Civil Engineering	Constructive geometry – 2/2
Brno University of Technology - Faculty of Civil Engineering	Basics of descriptive geometry – 2/0 Descriptive geometry – 2/2
University of West Bohemia in Plzeň - Faculty of Applied Sciences	Descriptive geometry I – 2/2 Descriptive geometry II – 2/1
Technical University of Ostrava Faculty of Civil Engineering	Descriptive geometry – 2/2
Architecture	
Czech Technical University in Prague Faculty of Civil Engineering	Constructive geometry A – 2/2
Czech Technical University in Prague Faculty of Architecture	Descriptive geometry I – 2/2 Descriptive geometry II – 1/2
Brno University of Technology Faculty of Architecture	Mathematics and Geometry – 2/2
Technical University of Liberec The Faculty of Art and Architecture	Descriptive geometry I – 2/2 Descriptive geometry II – 2/2
Technical University of Ostrava Faculty of Civil Engineering	Descriptive geometry – 2/2
Mechanical Engineering	
Czech Technical University in Prague Faculty of Mechanical Engineering	Constructive geometry – 3/2
Brno University of Technology Faculty of Mechanical Engineering	Computer Geometry and Graphics - 2/2
University of West Bohemia in Plzeň - Faculty of Mechanical Engineering	Geometry I – 2/2 Geometry II – 2/1
Technical University of Liberec - Faculty of Mechanical Engineering	Constructive geometry – 2/2
Technical University of Ostrava - Faculty of Mechanical Engineering	Constructive geometry – 2/2

Table 3: Geometric topics at technical universities in the Czech Republic

Czech Technical University in Prague
Faculty of Civil Engineering
Constructive geometry – 2/2 *Year of study: 1, Semester: Winter*
Projection and its properties. Axonometry, solids in axonometry. Linear perspective. Constructive photogrammetry. Curves, description, parametrizations. Helix. Surfaces of revolution. Helical surfaces. Ruled surfaces. Quadrics. Hyperbolic paraboloid. Conoid, cylindroid. Using Rhino software for modelling of curves and surfaces.
Faculty of Architecture
Descriptive geometry I – 2/2 *Year of study: 1, Semester: Winter*
Topographical surfaces. Conic sections. Projections. Orthographic and Monge projection, axonometry. Algorithms for positional and metric properties. Intersection of plane and solids. Linear perspective. Photogrammetry.
Descriptive geometry II – 1/2 *Year of study: 1, Semester: Summer*
Shadows in parallel lightings. Curves. Helix. Surfaces of revolution. Helical surfaces. Ruled surfaces. Conoids.
Faculty of Mechanical Engineering
Constructive geometry – 3/2 *Year of study: 1, Semester: Winter*
Kinematic geometry. Projections. Orthographic and Monge projection, axonometry. Analytic geometry in E_3. Quadrics. Surfaces of revolution. Helix. Helical surfaces. Rotational and helical surfaces - equation, meridian, tangent plane, cuts, common curve. Envelope surfaces - characteristic, methods of tangent sphere and plane. Developable surface - classification, development.
University of West Bohemia in Plzeň
Faculty of Applied Sciences (Civil Engineering)
Descriptive geometry I – 2/2 *Year of study: 1, Semestr: Winter*
Conic sections (ellipse, parabola, hyperbola), focus based and projective properties. Axial affinity. Central collineation. Transformation of circle. Orthographic and Monge projection, axonometry. Algorithms for positional and metric properties. Projection of solids (prisms, pyramids, cylinders, cones, sphere). Topographical surfaces.

Descriptive geometry II – 2/1 *Year of study: 1, Semester: Summer*
Solids, intersection with plane and straight line, intersection of the solids. Central projection. Linear perspective. Surfaces of revolution. Helix. Helical surfaces. Ruled surfaces.
Faculty of Mechanical Engineering
Geometry I – 2/2 *Year of study: 1, Semester: Summer*
Polynoms. Matrix operations. Determinants. Systems of linear algebraic equations. Inverse of matrix. Methods of projection - classification, orthogonal projection of perpendicular lines. Monge projection. Axonometry. Vector algebra: scalar, vector and triple product.
Geometry II – 2/1 *Year of study: 2, Semester: Winter*
Transformations (coordinates and objects) in E_2 and E_3. Differential geometry of curves - mathematical representation, tangent vector, curvature. Curves - plane curves, helix. Differential geometry of surfaces - mathematical representation, tangent plane, curvatures (Gauss curvature). Surfaces - classification, properties of surfaces. Rotational and helix surfaces - equation, meridian, tangent plane, cuts, common curve. Envelope surfaces - characteristic, equation, methods of tangent sphere and plane. Developable surface - classification, development. Quadrics - equation, signature, classification, split of common curve.
Brno University of Technology
Faculty of Civil Engineering
Basics of descriptive geometry – 2/0 (v) *Year of study: 1, Semester: Winter*
Euclidean constructions in plane, identical and similarity transforms in plane, ellipse by focus properties, the principles of perspective affinity, perspective collineation, using such properties in solving problems, basics of projection: Monge`s, orthogonal axonometry. 3D visualization and solve simple 3D problems, display simple geometric solids and surfaces in each type of projections.
Descriptive geometry – 2/2 *Year of study: 1, Semester: Summer*
Conics, focal properties, principles of perspective colineation and the basics of projection: Monge`s projection, orthogonal axonometry, and linear perspective. 3D visualization, solve simple 3D problems, display simple geometric bodies and surfaces in each type of projection, their section with a plane and intersections with a straight line. Draw a building in linear perspective. Curves and surfaces, construct a helix, orthogonal closed rule

right helical surface. Warped surfaces, construct a hyperbolic paraboloid, circle and parabolic conoid.
Faculty of Mechanical Engineering
Computer Geometry and Graphics -2/2 *Year of study: 2, Semester: Winter*
Euclidean space, topologic dimension, curve, surface, solid. Projective space, division ratio and cross ratio, projection. Basic mappings in plane and space, their analytic representation (rotation, translation, axis and central symmetry, homothety), analytic representation of parallel and central projection). Analytic curves, Point function, tangent and normal of curve, curvature. Analytic surfaces, isolines, tangent plane, normal, normal and Gaussian curvature (basic information). Focal and projective properties of conics, circle-ellipse affinity. Triangle, stripe and Rytz construction. Curve representation in CAD systems, linear point combination, control points. Bezier curves, B-spline curves and surfaces, NURBS curves. Fundamentals of kinematic plane geometry (motion, fixed and moving centrode, circle arc rectification, rolling motion, cycloid and involute curve - synthetic and analytic construction, animation principle, software modeling). Elementary surfaces and solids (prism, pyramid, cylinder, cone, sphere) Monge projection (MP) and orthogonal axonometry (OA), NURBS surfaces, NURBS representation of elementary curves and surfaces. Slices of solids, the intersection of line and solid, intersection of solids - Monge's projection and axonometry solutions. Helix, analytic representation, MP and OA projection. Methods of surface generation in graphic systems. Basic generating principles. Developable surfaces (cylindric and conic surface, curve tangent surface, transition surfaces). Undevelopable surfaces (conoid, cranc mechanism surface, oblique transition surface) - analytic representation, computer modeling. Rotation surfaces (torus, rotation quadric) - Monge's projection and axonometry - analytic representation, computer modelling Helical surfaces, cyclic and linear surfaces, - Monge's projection and axonometry, analytic representation, computer modeling. Hausdorff dimension, fractal. Self-similarity and self-affinity, random walk method, midpoint method, L-systems. Lighting of elementar solids, lighting models in computer graphics, Ray Tracing, Ray Casting.
Technical University of Liberec
The Faculty of Art and Architecture
Descriptive geometry I – 2/2 *Year of study: 1, Semester: Winter*
Focal properties of conics. Affinity. Ellipse in affinity. Perspective collineation. Conics in perspective collineation. Monge projection - position

problems. Monge projection - metric problems. Monge projection - image of a circle, projection of solids. Orthogonal axonometry - position problems. Orthogonal axonometry - metric problems. Orthogonal axonometry - image of a circle, projection of solids. Oblique parallel projection, military perspective projection. Plane section of solids.
Descriptive geometry II – 2/2 *Year of study: 1, Semester: Summer*
Shadows in parallel lightings. Lighting of surfaces of revolution (cylindrical, conical, spheres). Central projection - introduction. Linear perspective as a special case of central projection. Simple methods of perspective. Free perspective, vanishing points. Image of plane curves, sphere. 3-point perspective. Curves, basic properties; tangent, normal, point of inflection. Helix. Surfaces of revolution, basic properties. Plane section and intersection of surfaces of revolution. Ruled surfaces, basic properties. Helical surfaces, basic properties. Cyclic helicoids. Ruled helicoids.
Faculty of Mechanical Engineering
Constructive geometry – 2/2 *Year of study: 1, Semester: Winter*
Principles of Monge projections. Projection of points, straight lines, planes. Point of intersection of a straight line and a plane, parallel and intersecting planes. (Plane section of prisms and pyramids.) Straight line perpendicular to a plane, distance of linear objects. Rotation of a plane into a projection plane. Orthogonal projection of a circle. Elementary solids in general position, contour lines visibility. Analytic geometry v E3. Vectors, coordinates of vectors and points. Parametric equation of a straight line and a plane. General equation of a plane. Position and metric problems in E3. Vector function of one real variable. Definition and equation of a curve, tangent. Accompanying trihedron , flection and torsion. Helix. Equation, basic properties. Constructive problems. Surface, curves on a surface, tangent plane. Surface of revolution, meridian, tangent plane. Constructive problems, plane section, intersection. Helicoids. Definition, basic properties and constructive problems. Cyclic helicoids. Ruled helicoids.
Technical University of Ostrava
Faculty of Civil Engineering
Descriptive geometry – 2/2 *Year of study: 1, Semestr: Winter*
The basic properties of the projection. Central collineation, perspective affinity. The mapping projection, the Monge's projection, the orthogonal axonometry. Elementary surfaces and solid. Circular helix and moving trihedral. Surfaces of revolution, quadrics of revolution. The ruled surfaces, the developable and especially the skew ruled surfaces. Spiral surfaces.

Faculty of Mechanical Engineering
Constructive geometry – 2/2 *Year of study: 1, Semester: Summer*
Affinity. Monge projection. Orthogonal axonometry. Orthogonal projection of circle. Elementary surfaces and solid. Perspective collineation. Helix, helical surfaces. Surface of revolution, meridian, tangent plane. Ruled surfaces.

Geometry Education at Polish Universities of Technology

Monika Sroka-Bizoń

Polish Society for Geometry and Engineering Graphics
Krzywoustego 7, 44-100 Gliwice, Poland
email: monika.sroka-bizon@polsl.pl

The years 1999 and 2005 were crucial for the Polish education system. In 1999, a reform changing the system of education was introduced. The 8– year period of primary school was changed into 6– year primary schools and 3– year middle schools; the upper secondary education, earlier called “post-primary”, was renamed as “post-middle education”, and included 2– year and 3 – year basic vocational schools 3– year general and specialised secondary schools, and 4– year technical schools. The curriculums were substituted by core curriculums. The latest core curriculum for general education, changed four times since 1999, was introduced by the Regulation of 23 December 2008 by the Minister of National Education. In 2005 the Act on Higher Education introduced the following terms to the higher education system: areas of education, learning outcomes, National Qualifications Framework for Higher Education, curriculums based on learning outcomes. The Act definitely stated that the results of the secondary school exit examination are the basis for the admission to a university. Therefore, in accordance with the legislation, general education is a basis for education in a particular area of study. The table 1. presents the issues of the core curricula for mathematical education, especially geometric issues of the core. The table shows the range of the geometrical knowledge that students should have before beginning their studies. The table 2. presents the issues of the mathematical education which is realised at the universities of technology (on the example of the Civil Engineering course at the Faculty of Civil Engineering of the Silesian University of Technology). The table shows the range of the geometrical knowledge that students should have after their studies. The most important for universities’ level of mathematical education is to formulate curriculums for Mathematics and Descriptive Geometry as a continuation of general education at the level of higher education.

Table 1. Regulation of the Minister of National Education of 23 December 2008. on core curricula for pre-school education and general education in particular types of schools, published on 15 January 2009., Journal of Laws No. 4, pos. 17

IV stage of education - high school (young people aged 15/16 - 17/18)	
basic range	extended range
Planimetry	
▪ uses the relationship between central angle and inscribed angle ▪ uses the property tangent to the circle and properties of tangent circles ▪ recognize similar triangles and uses (also in practical contexts) of similarity of triangles ▪ uses the property trigonometric functions in easy geometric calculations, including the formula for area of a triangle acute-angled of data the two sides and the angle between them	Basic range, and moreover ▪ theorem applies characterizing the quadrangles inscribed in a circle and squares described on the circle ▪ apply the theorem of Thales and the reverse to Thales' Theorem for calculating and determining the lengths of parallelism ▪ is able to find images of some geometric figures in the homothety (segment, triangle, square, etc.). ▪ recognize the figures similar and homothety; uses (also in practical contexts) their properties ▪ is able to find measuring compounds in flat figures using the theorem of sines and cosines theorems
Cartesian geometry on the plane	
▪ Determine the equation straight through two data points (in the form of directional or general ▪ explores the parallelism and squareness of simple equations based on their direction ▪ determines the equation of a line that is parallel or perpendicular to the straight	Basic range, and moreover ▪ graphically interprets a linear inequality with two unknowns and systems such inequality ▪ explores the parallelism and squareness of simple equations based on their general ▪ determines the equation of a line that is parallel or perpendicular to the line given in the form of

basic range	extended range
line as the direction and passes through the center ▪ calculate the coordinates of the point of intersection of two lines ▪ determines the coordinates of the center section ▪ calculates the distance between two points ▪ is able to find images of some of geometrical figures (point, line, segment, circle, triangle, etc.) in axial symmetry about an axis of coordinate system and central of the centre of it	general and passes through the point ▪ calculates the distance from a point to a line ▪ uses the equation of the circle (x) 2+ (Y-b) 2 = r2 and describes the wheels by inequality ▪ designate points of the joint line and circle ▪ calculate the coordinates and the length of the vector; adds and subtracts vectors and multiplies them by number; interprets geometrically operations on vectors ▪ vectors are used to describe the displacement graph of the function
Stereometry	
▪ recognizes prisms and pyramids angles between segments (eg. between the edges, edges and diagonals, etc.), calculates measures of these angles ▪ recognizes prisms and pyramids angle between segments and planes (between the edges and the walls, diagonals and walls) calculates the measurement of these angles ▪ recognizes in cylinders and cones angle between sections and the angle between segments and planes (eg. a cone angle, the angle between a, and the base) are calculated measure of these angles ▪ recognizes prisms and pyramids angles between the walls	Basic range, and moreover ▪ specifies the shape of the cross-section of the sphere by plane ▪ specifies the shape of the cross-section of the pyramid or the prism by the plane

basic range	extended range
▪ specifies the shape of the a cross-section of the cuboid by a plane ▪ used trigonometry to calculate lengths, measure angles, surface areas and volumes	

Table 2. Geometrical issues of mathematical education at Polish universities of technology

I stage of studies (bachelor) – universities of technology (young people aged 18/19 - 21/22) – on the example of Civil Engineering Course at Silesian University of Technology	
Mathematics (2 semesters) (geometrical issues)	Descriptive Geometry (1 semester)
▪ basics of vector calculus in 3-dimensional space ▪ line and plane in 3-dimensional space ▪ derivative and its geometric interpretation ▪ curve sketching (asymptotes, monotonicity, extremes, concavity, points of inflection) ▪ definite integrals and its geometric and physical applications	▪ parallel projection and its properties ▪ parallel projection ▪ orthogonal projection ▪ Monge'an projection ▪ point, line and plane in space ▪ spatial relations ▪ common elements ▪ perpendicular elements ▪ curves ▪ polyhedrons – sections, developments ▪ surfaces – sphere, cones, cylinders ▪ Catalan's surfaces ▪ tophographic projections

Constructive Geometry at some Hungarian universities

Péter Körtesi, László Németh, Ildikó Papp

Hungarian Society for Geometry and Graphics

Abstract. Spatial geometrical knowledge and spatial ability are of basic importance for engineers. Constructive Geometry is the subject that is the best for developing this kind of knowledge. In this article we show the curriculum of Geometry in the Hungarian secondary schools, then we give examples of syllabuses of Mathematics and Constructive Geometry at some Hungarian universities.

Keywords: Education of Mathematics, Education of Constructive Geometry, Education of Descriptive Geometry.

1 Introduction

We can agree that a good spatial ability is essential not only for engineers but for everybody in almost all parts of everyday life. We can think of walking in a forest, or trying to find our way among the shelves of a huge hypermarket besides of planning a new object. For engineers and artists, in addition to spatial ability, the knowledge of geometric solids, their properties, connections and correct visualisations is also of primary importance. Geometry is the base for acquiring all these skills.

Teachers of Mathematics, especially those of all the different parts of geometry, have the task and responsibility of passing this knowledge, adjusted to the requirements of the 21st century as widely as possible. This process happens mainly during the university years, in higher education, within the courses such as Geometry, Descriptive Geometry, Constructive Geometry, Computer Geometry and Engineering Drawing. Out of all these subjects and topics, perhaps it is the expression Constructive Geometry that covers best the part of geometry that deals with spatial solids and their visualisations as well as the defining of new objects. In Hungary, as well as in any other country of the world, teachers find constructive and descriptive geometrical training really important. The development of computer CAD programmes does not eliminate the teaching of this knowledge, but it can determine new directions and new options in constructive geometry. Hungarian teachers of constructive geometry, both in secondary and in higher education, founded the Hungarian Society for Geometry and Graphics. They try to hold together and coordinate the education of geometry and to meet the expectations of our age.

In this article first we give a list of topics of geometry having been taught in the secondary schools in Hungary. Then we present the Mathematics syllabuses of some courses for engineers at the University of Miskolc. This list highlights which geometrical areas are essential for the students of engineers during their mathematical studies. Finally, we show examples for the curricula of Constructive Geometry courses at the University of Miskolc and University of West Hungary.

2 Geometry in secondary schools in Hungary

Foundation of Constructive Geometry starts in the elementary and secondary schools by the basics of planar and spatial geometry. The list[1] bellow gives the topics of geometry teaching built in the subject Mathematics in Hungarian secondary education.

Measurement. Measurement of Length, Measurement of Mass, Measurement of Capacity, Measurement of Time, The Clock, Timetables, Average Speed, The Arithmetic of Metric Quantities, Money.

Transformation Geometry. Introduction, Translation, Properties of Translation, Reflection, Reflection in the x-axis, Reflection in the y-axis, Reflection in the line $y = x$, Reflection in the line $y = -x$, Properties of Reflection about a Line, Symmetry, Reflection about a Point, Properties of Reflection about a Point, Rotation about a Point, Properties of Rotation about a Point, Finding the Centre of Rotation, Enlargement, Areas of Enlargements.

Symmetry, Patterns and Tessellation. Line Symmetry, Rotational Symmetry, Point Symmetry, Planes of Symmetry, Patterns, Tessellation.

Distance, Angles and Straight Lines. Definitions, Distance, Angles, Angular Measurement, Types of Angle, Angle between a Line and a Plane, Angle between Two Planes, Angle between Two Skew Lines, Properties of Angles and Straight Lines, Parallel Lines, Drawing Parallel Lines, Dividing a Straight Line into a Number of Equal Parts, The Intercept Theorem and its converse, Perpendicular Lines, Constructing Perpendiculars, Finding a Mirror Line, Constructing Angles, Bearings, Back Bearings.

Triangles. Types of Triangle, Angle Properties of Triangles, Standard Notation for a Triangle, Theorems Involving Triangles, Equilateral Triangle, Isosceles Triangle, Construction of Triangles, Construction of the Circumscribed Circle of a Triangle, Construction of the Inscribed Circle of a Triangle, Congruent Triangles, Similar Triangles, Areas of Similar Triangles.

Quadrilaterals and Other Polygons. Introduction, Classifying the Plane Quadrilaterals, Classifying the Trapezoids according to the Number of Pairs of Parallel Sides, Classifying the Plane Quadrilaterals according to the Number and position of sides of equal length, Other Polygons, Sum of the Interior Angles of a Polygon, Sum of the Interior Angles of a Convex

[1]Thanks for Wiersumné dr. Gyöngyösi Erika

Polygon, The Number of Diagonals of a Convex Polygon, Symmetry of Regular Polygons.

Circle. Introduction, Angles in Circles, Cyclic Quadrilaterals, Circumscribed Quadrilateral, Chords, Tangent Properties of a Circle, Constructing Tangents.

Locus of Points. Introduction, Standard Loci, Intersecting Loci, Three-Dimensional Loci.

Trigonometry. The Notation for a Right-Angled Triangle, The Trigonometric Ratios, Trigonometric Ratios for 30°, 60° and 45°, Complementary Angles, The Squares of the Trigonometric Ratios, Angle of Elevation, Altitude of the Sun, Angle of Depression, Solid Trigonometry, Calculating Bearings.

Vectors. Vector Quantities, Representing Vectors, Equal Vectors, Inverse Vectors, Sum of Vectors, Triangle Law The Properties of Vector Addition, Subtraction of Vectors, Multiplying by a Scalar, Distributive Law for Vectors, Base Vectors, The Absolute Value or Magnitude of a Vector, The Scalar Product of Vectors, Properties of Scalar Product, The Scalar Product of Two Vectors Described by Coordinates.

Sine and Cosine Rules. Trigonometric Ratios between 0° and 180°, The Solution of Triangles, The Sine Rule, Use of the Sine Rule to find the Radius of the Circumscribing Circle of a Triangle, The Cosine Rule, Trigonometric Functions, Trigonometric Equations.

Coordinate Geometry. Introduction, The Length of a Segment, The Mid-Point of a Segment, The Trisector Formula, The Coordinates of the Centroid in a Triangle, Graphs of Linear Equations, The Gradient of a Line, Equation of a Straight Line Derived from the Direction-Vector Form, Equation of a Straight Line Derived from the Normal-Vector Form, The Gradient form for an Equation of a Line, The Meaning of m and b in the Equation of a Straight Line, Experimental Data, Parallel Lines, Perpendicular Lines, Point of Intersection of Two Lines, The Distance between a Point and a Line and the Distance between two Lines, Equations of circles, The Circle and the Two-variable Quadratic Equation, The Common Point of a Straight Line and a Circle and that of two Circles, Tangent to a circle, Parabola, History of geometry.

Areas and Volumes. Perimeters and Areas of Planar Figures, Surface Areas and Volumes of Solid Figures, Similarity of Triangles, Similar Figures, Similar Solids.

3 Mathematics subjects at the University of Miskolc

(Department of Analysis)

Mathematics I: 1 semester, 5 lessons a week.

Complex numbers. Vector operations, Matrices and Determinants, Linear Systems, Linear Transformations, Base, Eigenvalues and Eigenvectors. Limits, Continuous Functions, Derivative, Derivative Formulas, Concavity, Maxima and Minima Problems, the Definite and Indefinite Integral, Techniques of Integration, Area of Plane Regions, Arc Length, Work, Energy and other applications.

Mathematics II: 1 semester, 5 lessons a week.

Parametric and polar coordinate curves. Derivatives of implicit functions, applications of derivatives for parametric and polar coordinate curves. Numerical and power series, convergence criteriae, Taylor polynomials and series. Ordinary differential equations (separable, first order linear DE-s, incomplete second order DE-s, constant coefficient DE-s, Euler type DE-s. Coordinates in 3 dimensions, surfaces and curves in the space. Definition, limit, continuity and partial differentiation of 2 and 3 variable functions. Gradient, normal, tangent plane, Taylor polynomial for 2 and 3 variable functions. Integration of 2 and 3 variable functions, cylindrical and spherical coordinates, change of variables, Jacobian, applications. Serret-Frenet trihedron, Integrals in vector spaces, applications in geometry and mechanics. Theorem of Gauss-Ostrodgradskij, formulae of Green-and Stokes.

Differential Equations: 1 semester, 4 lessons a week.

Origin and classifications of differential equations. Equations of first order: variable separable and reduction, linear equations and those reducible to that form. Linear equations with constant coefficients. Variation of parameters. Euler equation. System of linear differential equations. Second order linear differential equations: Homogeneous and non-homogeneous linear equations, Euler-Cauchy equations.

Complex functions: 1 semester, 2 lessons a week.

Complex algebra. Complex function. Elementary complex functions, Riemann surfaces. Limits and continuity of complex integration and Cauchy's theorem. Cauchy's integral formula. Laurent's series. The residue theorem. The unilateral Laplace-transforms and its properties. Evaluations of inverse transforms. Applications.

Mathematics for Economic Analysis I.: 1 semester, 4 lessons a week.

Introduction to the basic concepts of calculus and their applications. Functions, derivatives, and limits; the definite integral. Techniques of integration, applications of integration. Introduction to single variable probability. Random variables. Distributions.

4 Constructive Geometry at the University of Debrecen

Applied constructive geometry appears in engineering education at the University of Debrecen. The Faculty of Engineering and the Faculty of Informatics are involved in these courses, which include Descriptive Geometry, Technical drawing and Computer engineering. Previously, there used to be a course on geometry before the computer graphics studies, but some years ago it was removed from the curriculum of Computer engineering. Experience shows that students have serious geometric deficiencies in their graphic studies. The Descriptive geomerty and Technical drawing courses intend to help precise work in problem solving and to prepare computer-aided design. Now we present our currently running courses.

4.1 Civil Engineering

Descriptive Geometry: 1 semester, 4 lessons a week.

The Monge representation: making of images, methods of projection, the Monge image plane system, representation of spatial elements, reconstruction. The fundamentals of intersection: designing a stabbing point for normal and special cases, plane-plane intersection. The fundamental tasks of metricity: perpendicularity, distance and angle tasks, rotation of a plane into an image plane, image plane transformations, designing a graphical picture by image plane transformation, visibility. Depiction of planar polygons and circles on a given plane. Polyhedrons (prism, pyramid): their representation, their plane intersection, their interpenetration. Base of axonometric and central projections.

4.2 Industrial Engineering, Architectural Engineering

Descriptive Geometry 1.: 1 semester, 4 lessons a week.

The Monge representation: making of images, methods of projection, the Monge image plane system, representation of spatial elements, reconstruction. The fundamentals of intersection: designing a stabbing point for normal and special cases, planes and plane-plane intersection. The fundamental tasks of metricity: perpendicularity, distance and angle tasks, rotation of a plane into an image plane, image plane transformations, designing a graphical picture by image plane transformation, visibility. Depiction of planar polygons and circles on a given plane. Polyhedrons (prism, pyramid): their representation, their plane intersection, their interpenetration. Construction of roof profiles.

4.3 Industrial Engineering, Architectural Engineering

Descriptive geometry 2.: 1 semester, 3 lessons a week.

Surfaces (cone, cylinder, sphere, torus): construction of surfaces, representation of surfaces, their plane intersection, interpenetration of surfaces. Construction of shadows. Representation of polyderons, surfaces and shadows in axonometric and central projections.

4.4 Mechanical Engineering, Mechatronical Engineering

Technical drawing 1.: 1 semester, 3 lessons a week.

The Monge representation: making of images, methods of projection, the Monge image plane system, representation of spatial elements, reconstruction. The fundamentals of intersection: designing a stabbing point for normal and special cases, planes and plane-plane intersection. The fundamental tasks of metricity: perpendicularity, distance and angle tasks, rotation of a plane into an image plane, image plane transformations, designing a graphical picture by image plane transformation, visibility. Depiction of planar polygons and circles on a given plane. Polyhedrons (prism, pyramid): their representation, their plane intersection, their interpenetration. Surfaces (cone, cylinder, sphere, torus): construction of surfaces, representation of surfaces, their plane intersection, interpenetration of surfaces.

4.5 Computer Science, Computer Science Engineering

Computer engineering: 1 semester, 2 lessons a week.

Basic concepts of computer engineering, user interface of software *SolidWorks*. Basic operations: 2D-3D sketches, relations, dimensions. Features: extruded, revolved, swept, loft boss/base, assembly of parts, operations of parts. Modification of models: fillet, chamfer, linear and circular pattern, mirror. Display options, generation 2D engineering drawing.

5 Constructive geometry at the University of West Hungary

There is education of engineers and artists in two faculties of the University of West Hungary, the Faculty of Forestry and Simonyi Károly Faculty of Engineering, Wood Sciences and Applied Arts. The students have Descriptive Geometry, Engineering Drawing and Computer Geometry courses, where they can study the appropriate constructive geometry knowledge.

5.1 Forestry and Environment Protection Engineering

Descriptive geometry and Engineering drawing: 1 semester, 4 lessons a week.

Basics of parallel and orthogonal projections (drawings with cotes, Monge-projection, axonometry). Views, auxiliary views, drawing view transformation. Curves and surfaces, their descriptions, section views, intersections and rectifications. Contour-maps, surfaces with contour-lines, sections, pads, drawings on contour-maps. Central projection and perspective. Basics of engineering drawing. Views, section views, sections. Dimensioning rules. Drawing standards.

5.2 Wood Industrial Engineering and Industrial Design Engineering

Descriptive geometry, BSc: 1 semester, 3 lessons a week.

Spatial relationships between points, lines, and planes. Definition and properties of parallel projection. The bases of the Monge- and axonometric projections. The description of common surfaces often used in technical drawings (pyramid, prism, cylinder, cone, sphere), their plane sections and intersections. Central- and perspective projection.

Computer drawing, MSc: 1 semester, 4 lessons a week.

Solid modelling, representing solids, boundary representations, Euler operators. Approximations, spline method. Bézier and B-spline curves, nonuniform rational B-spline (NURBS) and their properties, types and generalisations. Parametrizations. Surface modelling. Ruled surfaces, Coon's patches, Bézier, B-spline and rational B-spline surfaces and their properties, types and generalisations. Shape modifications. Complex surface representations and modifying. Transformations, projections. Prepare several 3D-model with software *AutoCAD*.

5.3 Architectural Arts, Product Design and Graphic Design

Descriptive geometry, BSc: 2 semesters, 2 lessons a week.

Basic ideas and theorems of Euclidean geometry. Euclidean construction. Planar and spatial shapes (polygons, curves, polyhedra) and their modelling. Parallel and central projections. Monge's projection. Projection plane's transformation. Incidental and metrical problems. Description of spatial elements and polyhedra. Plane section and intersection of polyhedra. Regular polyhedra. Base of axonometric projection. Axonometric description of polyhedra. Construction of shadow.

Curves and surfaces, their classifications and modelling. Descriptions of curves and surfaces in Monge's projection and axonometric projection. Plane sections and intersections of surfaces. Central- and bi-central projections, anaglyph pictures. Perspective projection and descriptive methods. Construction of shadows. Descriptions of complex surfaces and solids.

Visegrad Fund