32nd Symposium
on Computer Geometry SCG´2023

43rd Conference
on Geometry and Graphics

9th SLOVAK – CZECH CONFERENCE ON GEOMETRY AND GRAPHICS

Book of Abstracts

11. – 14. 9. 2023, Kremnica, SR

Slovak Society for Geometry and Graphics

Czech Society for Geometry and Graphics

Conference Organization Committee

Velichová Daniela – coordination and programme

Richtáriková Daniela – registration and administrative issues

Szarková Dagmar – webpage and database

Conference Scientific Committee

Programme

MONDAY, 11.9.2023 Arrival and registration of participants		
17^{00}	Conference opening	
	Chairs: Daniela Velichová - Miroslav Lávička	
17^{10}- 17^{20}	Emil Molnár	The Kárteszi point of a triangle by the absolute sine theorem of János Bolyai. To the Memory of my Mentor Professor Ferenc Kárteszi (1907-1989)
17^{20}- 18^{20}	Zbyněk Šír	Seven versions of de Casteljau algorithm
18^{30}	**Dinner**	
19^{00}	**Get – together party**	

TUESDAY, 12.9.2023		
8^{00}	**Breakfast**	
	Chairwoman: Daniela Velichová	
9^{30}- 10^{30}	Demeter Krupka	Geometric modeling: Variational principle
10^{30}- 11^{00}	**Coffee break**	
11^{00}- 11^{20}	Gunter Weiss	3D-versions of theorems related to Miquel´s Theorem
11^{20}- 11^{40}	Boris Odehnal	Variations on Fregier´s Theorem
11^{40}- 12^{00}	Miroslav Lávička	Euclidean symmetries of implicit surfaces in 3D
12^{00}- 12^{20}	Ján Brajerčík	Geometry of Schwarzschild spacetimes
12^{20}- 14^{00}	**Lunch**	

TUESDAY, 12.9.2023

	Chairman: Zbyněk Šír	
14^{00}- 14^{20}	Jan Vršek	On computation of symmetries of algebraic objects
14^{20}- 14^{40}	Pavel Chalmovianský	Triangulation of implicit surfaces with singularities
14^{40}- 15^{00}	Martin Čavarga	Single-parameter mesh primitive counting formulas for subdivision surfaces
15^{00}- 15^{30}	**Coffee break**	
	Chairwoman: Šárka Voráčová	
15^{30}- 15^{50}	Marie Koktavá	Ames´s illusions
15^{50}- 16^{10}	Ivana Linkeová	Parametric CAD model of axial blood pumps components
16^{10}- 16^{30}	Marta Hlavová	On the use of Bézier bicubic surface for shape reconstruction
16^{30}- 16^{50}	Jakub Müller	Impact of surface continuity on 3D model
16^{50}- 17^{10}	**Short break**	
17^{10}- 17^{30}	Jaroslav Cibulka	VTOL propeller development in start-up environment
17^{30}- 17^{50}	Bronek Pabich	The family of uniform polyhedra 3D Printing as tools for teaching 3D Geometry
17^{50}- 18^{10}	Alexej Kolcun	Are natural fractals really fractals?
18^{10}- 18^{30}	Jakub Řada	Selected geometric constructions and proofs
18^{30}	**Dinner**	
19^{30}	**Meetings of societies SSGG, ČSGG**	

WEDNESDAY, 13.9.2023

8^{00}	**Breakfast**	
	Chairman: Pavel Chalmovianský	
9^{30}- 10^{30}	Domen Kušar	Descriptive geometry in Slovenia, decline or oportunity
10^{30}- 11^{00}	**Coffee break**	
11^{00}- 11^{20}	Juliana Beganová, Margita Vajsáblová	Dynamická učebnica deskriptívnej geometrie pre stavebné odbory
11^{20}- 11^{40}	Stanislava Čečáková Dana Kolářová	Sliceforms in the education of architecture students
11^{40}- 12^{00}	Manaswita Das	On multicriteria triangulations
	Tatiana Rückschlossová	Priamkové plochy - tlačené 3D modely vytvorené v prostredí Rhinoceros
12^{10}- 13^{00}	**Lunch**	
13^{30}- 18^{00}	**Excursion**	
19^{00}	**Conference Dinner**	

THURSDAY, 14.9.2023

8^{00}	**Breakfast**	
	Chairman: Alexej Kolcun	
9^{30}- 9^{50}	Šárka Voráčová	Application of Euler path
9^{50}- 10^{10}	Věra Ferdiánová Michaela Holešová	Ukázky konstrukcí oválů v ostravské architektuře 19. století
10^{10}- 10^{30}	Patrik Peška Lenka Vítková	Konstrukce racionálních čísel pomocí origami
10^{30}- 11^{00}	**Coffee break**	
	Chairwoman: Margita Vajsáblová	
11^{00}- 11^{20}	Alice Králová	Parametrizace vybraných šroubových ploch a jejich zobrazení v programech GeoGebra a Maple
11^{20}- 11^{40}	Michal Zamboj	Calculating on a parabola
11^{40}- 12^{00}	Marcel Makovník	Subdivision and conchoids
12^{00}- 12^{20}	Bosáková Adriana	Contribution of a tangent of multiplicity one to the intersection multiplicity of two plane curves
12^{20}- 12^{30}	Ján Gunčaga, Štefan Tkačik	Osobnosti slovenskej matematiky – inšpirácia a životné vzory pre budúce generácie
13^{00}	**Lunch**	
	Departure	

Contents

Seven versions of de Casteljau algorithm
Sedm tváři de Casteljau algoritmu

ŠÍR Zbyněk

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In memory of Paul de Casteljau (1930-2022) we will present various aspects of his famous algorithm. First, we will outline the original affine version, including a short presentation of the original French paper (de Casteljau 1959). Then we will analyze the projective version (Farin 1983) and will present two interpretations within the classical projective geometry. We will also study alternative versions of the rational algorithm based on the factorization of the curve denominator (Šír and Jüttler 2015). Finally, we will show the generalizations of the de Casteljau algorithm to the case of spline curves, complex curves a various surfaces.

Inspirováni vzpomínkou na Paula de Casteljau (1930-2022) se budeme věnovat různým aspektům jeho slavného algoritmu. Nejprve připomeneme vlastnosti původní afinní verze včetně náhledu na původní článek (de Casteljau 1959).
Poté přestavíme některé z velkého množství zobecnění tohoto algoritmu.
Podrobně vysvětlíme jeho racionální verzi (Farin 1983) a vysvětlíme dva pohledy jak chápat jeho projektivní aspekty.
Představíme i jeho alternativní verze založené na rozkladu jmenovatele (Šír and Jüttler 2015). Dále ukážeme jeho zobecnění pro splajnové křivky, křivky v komplexním oboru a různé typy ploch.

Geometrické modelovanie: Variačný princíp

Geometric modeling: Variational principle

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Medzi aktuálne problémy súčasného výskumu v diferenciálnej geometrii patria jej aplikácie v teórii riadenia (regulácie) mechanických systémov. Prednáška je zameraná na úvod do geometrickej teórie riadenia (geometrická robotika). Zahrnuje metódu modelovania matematických a fyzikálnych procesov a problémov stability v technických vedách pomocou variačných princípov. Sú tiež diskutované nové možnosti konštrukcie variačného popisu (pohybových) rovníc a možnosti jeho rozšírenia v geometrickej robotike.
Prednáška nevyžaduje špeciálne matematické znalosti

The current research in differential geometry includes applications of geometric methods in control of the motion of mechanical systems. In this presentation we discuss elementary ideas of the geometric control theory (geometric robotics) and introduce a method of mathematical modeling of processes and stability problems in physics and engineering using variational principles. New possibilities of introducing the control parameters and extensions of the variational description of equations of the motion are also discussed.
The lecture does not require special mathematical knowledge.

Descriptive geometry in Slovenia, decline or oportunity

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Descriptive geometry is a science that has been indispensable in all areas of technical education for many years.In addition to the way of representing spatial elements and their relationships on a two-dimensional medium, it also developed a spatial representation.We have been following the downward trend of spatial ability among students of the Faculty of Architecture of the University of Ljubljana since 1999.
With the advent of computers, especially computer graphics, the attitude towards descriptive geometry also changed.Since computers took over the presentation of space, it was no longer necessary. Therefore, the number of hours was reduced, and in many faculties, it was simply abolished or combined with other similar subjects.In Slovenia, this happened at many technical faculties.Part of the blame is also on the side of educators who did not understand the situation and the role that descriptive geometry can play and adapted the material to current needs or even upgraded it.
At the Faculty of Architecture of the University of Ljubljana, beside classical knowledge we included into the course computer programs and new teaching methods.Flexibility helped us to carry out the subject remotely without major problems during Covid.With the project of pilot updating the course using information and communication technology, we wanted to bring the course even closer to students and make it accessible at any time.However, it turned out that digitization also has its limitations, as pedagogical work also requires a personal approach and communication between the pedagogue/assistant and students.This means finding a balance between the use of modern techniques and "classical" pedagogical approaches with the aim of achieving the best possible knowledge and the greatest possible spatial representation that future architects will need in their work.

Dynamická učebnica deskriptívnej geometrie pre stavebné odbory

Dynamic university textbook of descriptive geometry for construction study programs

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Digitálna forma učebných materiálov dáva študentovi efektívnejšiu možnosť osvojenia geometrických pojmov a konštrukcií. Vysokoškolská učebnica „Deskriptívna geometria pre stavebné odbory“ bola publikovaná kolektívom autoriek v roku 2022 v elektronickej forme a je určená pre študentov študijných programov zameraných na staviteľstvo, architektúru, krajinné a konštrukcií pomocou 2D obrázkov, stereoskopických obrázkov, animácií a videí s ohľadom na typ použitých softvérových prostredí a nástrojov plánovanie, geodéziu a kartografiu. V príspevku ukážeme využitie rôznych foriem dynamických prezentácií geometrického učiva, ktoré zahrňujú teóriu a vizualizáciu pojmov. Uvedený prístup vo veľkej miere podporuje pochopenie a priestorovú predstavivosť študentov.

The digital form of study materials gives students a more effective way to learn geometric terms and constructions. The university textbook „Descriptive geometry for construction study programs” was published by a collective of authors in 2022 in electronic form and is suitable for students of study programs focused on civil engineering, architecture, landscape planning, geodesy and cartography. In the paper, we will show the use of various forms of dynamic presentations of geometrical content, which include theory and visualization of concepts and constructions using 2D images, stereoscopic images, animations and videos, taking into account the type of software environments and tools used. This approach greatly supports students' understanding and their spatial imagination.

Contribution of a tangent of multiplicity one to the intersection multiplicity of two plane curves

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Intersection multiplicity of two plane curves F and G at some point P has a well-known property of $I_P(F,G) \geq mn$, where m and n are the multiplicities of the point P on the curves F and G respectively. To each common tangent of F and G at P can be assigned a nonnegative integer, a number equal to its contribution to the intersection multiplicity. This can be done via local investigation methods. The sum of these contribution numbers of all common tangents is equal to the remainder $R = I_P(F,G) - mn$, We investigate the values of this contribution number for common tangents of the multiplicity 1.

Geometry of the Schwarzschild spacetimes

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The Schwarzschild spacetime is a basic model of the classical general relativity theory, describing the gravitational field outside a spherical mass. This contribution is devoted to its geometric theory. In a broader sense, a Schwarzschild spacetime is a smooth manifold, endowed with an action of the special orthogonal group SO(3) and a Schwarzschild metric, which is an SO(3)-invariant metric field, satisfying the Einstein equations. Explicit formulas of all Schwarzschild metrics on four-dimensional manifold $\mathbf{R}\times(\mathbf{R}^3\setminus\{(0,0,0)\})$ are introduced in a sense of spherical charts. To find these formulas we first classify all metrics on $\mathbf{R}\times(\mathbf{R}^3\setminus\{(0,0,0)\})$ invariant with respect to the action of SO(3), time-translations, and time-reflection.
The resulting family of Schwarzschild metrics is parametrized by a function and two real parameters, the integration constants. For any Schwarzschild metric, one of the parameters determines a submanifold, where the metric is not defined, the Schwarzschild sphere. In particular, the family admits a global metric whose Schwarzschild sphere is empty.
By the winding mapping of the real line **R** onto the circle S^1, these results are transferred to the manifold $S^1\times(\mathbf{R}^3\setminus\{(0,0,0)\})$ which topologically differs from $\mathbf{R}\times(\mathbf{R}^3\setminus\{(0,0,0)\})$.
All our assertions are derived independently of the signature of the Schwarzschild metric; the signature can be chosen as an independent axiom.

VTOL propeller development in start-up environment

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City air mobility transport with vertical take-off and landing capabilities challenges engineers across development spectra. Fast prototyping of parametric propeller geometry will be introduced as well as start-up approach to new geometry requirements. Capabilities and deficiencies of open propeller geometry software will be compared to classical licensed software approach.

SLA 3D printer used as a suitable tool for basic geometry evaluation and aero-acoustic testing will be described.

Single-parameter mesh primitive counting formulas for subdivision surfaces

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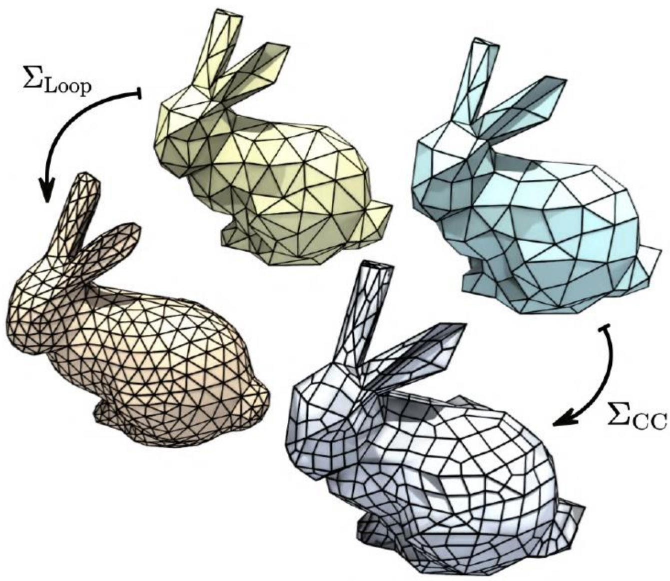

Methodology

Our approach begins with the formal definitions of a mesh, tessellation-changing operations, and subdivisions. Building on this foundation, we derive a new counting formula for Loop 4:1 subdivision in the context of triangle meshes. We extend this methodology to other subdivision techniques like Catmull-Clark.

1. **Formal Definitions:** Explanation of core terms to set the foundation for the methodology.
2. **Counting Formula for Loop 4:1:** We derive a formula, supported by rigorous proof, for estimating vertex density in Loop 4:1 subdivisions of triangle meshes.
3. **Catmull-Clark & Other Subdivisions:** Extension of the methodology to other common subdivision techniques.

Results and Performance

We present numerical and empirical results to demonstrate the efficacy of our proposed method in estimating vertex density and mesh primitive counts. Our approach is crucial in problems like Lagrangian shrink-wrapping. We also evaluate the possibility of efficient memory pre-allocation.

Sliceforms in the education of architecture students

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„Geometry is a part of mathematics that does not belong to mathematicians. It plays a role in all cultures in defining aesthetics from patterns through architecture to other aspects of day to day design. “
John Sharp

The presentation focuses on the use of sliceforms in the education of architecture students at CTU.

It shows not only historical models created via this method, but also the work of current students who used the latest technologies for their creation (CAD modelling, GeoGebra, laser cutting, 3D printing).

Sliceforms are useful for improving cross-subject relationships, most notably showing a close relationship between mathematics and descriptive geometry.

Exhibition of student’s models:

https://media.cvut.cz/cs/foto/20230517-slice-forms-fa

On multicriteria triangulations

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We survey the research of ´Multicriteria Optimized Triangulation´. This approach extends the classical Delaunay triangulation by incorporating optimization objectives such as edge lengths, angles, and even user-defined constraints. Multiple authors preferred stochastic methods mainly containing genetic optimization and edge flip. This approach proves useful in various applications, including computer graphics, mesh generation, and geographic information systems, where different factors need to be considered simultaneously for constructing accurate and adaptable geometric structures. Hence we will dive into the explorations done and results achieved in successive experiments afterwards, and propose the classification of various criteria. We present a unifying notation and discuss recent findings within the broader context of subgraphs of multicriteria triangulations of planar pointsets.

Ukázky konstrukcí oválů v ostravské architektuře 19. století

Examples of ovals in 19th-century architecture in Ostrava

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Výuka geometrie, zejména planimetrie na středních školách je zaměřena na základní konstrukce a využití shodností a podobností v konstrukčních úlohách. V nepovinně volitelných seminářích nebo na technicky orientovaných školách se představují konstrukční techniky pro kuželosečky, ale konstrukce oválů bývá na pozadí. Studenti se s problematikou setkávají povětšinou až na vysoké škole. Příspěvek bude ilustrovat různé konstrukce oválů na původní technické dokumentaci budov v Moravské Ostravě 19. století a přiblížení této problematiky studentům.

The teaching of geometry, especially planimetry, in high schools focuses on basic constructions and the use of congruences and similarities in construction problems. In optional seminars or in technically oriented schools, construction techniques for conics are introduced, but the construction of ovals tends to be in the background. Students are usually not exposed to the problem until they are at university. The paper will illustrate various constructions of ovals on the original technical documentation (blueprints) of buildings in 19th century Moravian Ostrava, and it will show how the issue can be introduced to students.

Osobnosti slovenskej matematiky - inšpirácia a životné vzory pre budúce generácie

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V rámci projektu KEGA č. 004KU-4/2022 "Osobnosti slovenskej matematiky II" pokračujeme v úspešnej edícií Osobností slovenskej matematiky. Projekt je zameraný na zachytenie života a diela významných osobností slovenskej matematiky v 20. storočí ich vlastnými slovami. V rámci príspevku chceme prezentovať spracované materiály o doc. RNDr. Daniele Velichovej, CSc. Je významnou osobnosťou v oblasti geometrie a geometrického modelovania útvarov. Venuje sa aj počítačovej grafike, e-learningu vo vyučovaní matematických disciplín. Jej príklad môže slúžiť ako inšpirácia pre mladých a začínajúcich vedcov a ukazuje, že aj matematika môže byť zaujímavou a nádhernou disciplínou prístupnou pre mladých.

On the use of Bézier bicubic surface for shape reconstruction

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How to exactly interpolate quadrilateral grid of points? In this article, some possibilities of using a Bézier bicubic patch for interpolation surface modelling are presented.

Ames´s illusions

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Last year, as usual, we participated in the Czech European Researchers´ Night at Mendelu. Since the theme was “With all the senses”, as geometers we focused on vision. We created two of Adelbert Ames Jr.´s most famous illusions, namely the Ames Room and the Ames Window. In this paper we will describe how these illusions work and how we recreated them.

Are natural fractals really fractals?

KOLCUN Alexej

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Self-similar structures attract our attention for a very long time. In the contribution, we will focus on constructions that are inspired by the Koch curve. We will point out the problematic nature of the BCM (Box Counting Method).

Triangulation of implicit surfaces with singularities

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A zero set of a suitable function is visualized using marching triangle technique with adaptation by Gauss curvature. Such an approach cannot be used in presence of singularities at the surface. We consider a description of ADE classes of singularities on a surface. Using suitable parameters, a neighborhood of a singularity is triangulated and the result is glued to the triangulation of the regular part of the surface. The algorithm is able to deal with bounding polyhedra as well. Examples of the approach are shown in detail.

Parametrizace vybraných šroubových ploch a jejich zobrazení v programech GeoGebra a Maple

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V příspěvku ukáži odvození parametrických rovnic pro některé přímkové a cyklické šroubové plochy. Tyto parametrizace umožňují vytvořit vizualizace těchto ploch v programech GeoGebra a Maple prostřednictvím statických modelů nebo animací. Oba programy jsou porovnány z hlediska funkčnosti pro zobrazení ploch, náročnosti jejich použití a kvality grafických výstupů.

Euclidean symmetries of implicit surfaces in 3D

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We present a complete algorithm to compute the rotational, axial, reflectional and central symmetries of an algebraic surface defined by means of its implicit equation. The functionality of the proposed algorithm is illustrated with several examples.

Parametric CAD model of axial blood pumps components

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The paper presents the development of a fully parametric CAD model of axial blood pumps components in Grasshopper, a graphical algorithm editor integrated in Rhinoceros. Since scientific papers report very little (if any) geometric information about the pumps, a special reverse engineering process was used to develop the parametric CAD model.

Conchoids and subdivision

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We discuss the refinement of a planar polyline using conchoids. For each segment of the polyline, we construct a conchoid, which interpolates its endpoints. This is achieved by choosing a feasible coordinate system, scaled by the input global parameter. Then, we choose new points from the interpolating conchoid, symmetrically to the horizontal axis. Afterwards, we transform the new pair of points to the original coordinate system. The refined polyline is obtained in the "corner-cutting" fashion, i. e. by joining the subsequent pairs of new points. The process of refinement may be applied repeatedly to achieve the desired level of detail. The proposed refinement scheme is approximating and non-linear. We provide several examples that demonstrate the behaviour of the refinement. Also, we inspect on the influence of the value of the global parameter. For the specific value, we obtain the well-known Chaikin´ s algorithm.

The Kárteszi points of a triangle, via three reflections theorem and geometric algebra

To memory of my professor and doctor father, Ferenc Kárteszi (1907-1989)

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Let us recall a **well-known school task:** *In the* (Euclidean $\mathbf{E}^2$) *plane of a triangle ABC we draw regular triangles outward on sides of ABC, say ABC¯, BCA¯, CAB¯, respectively. Prove that the segments AA¯, BB¯, CC¯ intersect each other in a point K*, *that is the isogonal point of ABC and the distance sum AK + BK + CK is minimal for K among all points of the plane*.

Professor Kárteszi noticed that instead of regular triangles we can draw isosceles ones with all equal base angles, and the above *K* (called *Kárteszi point*) exists also in the Bolyai–Lobachevsky hyperbolic plane $\mathbf{H}^2$ (in the sphere $\mathbf{S}^2$ as well, (see also Kálmán, 1989 and Sect. 2), the orthocentre, barycentre are specific cases. There is **a more general extremum problem** of (Yaglom, 1968, problem **83**, with modified notation):

In the plane ($\mathbf{E}^2$) *of a given triangle ABC find a point K such that the quantity* α*KA* + β*KB* + γ*KC, where* α, β, γ *are given positive numbers, has the smallest possible value.*

This problem leads to a more general triangle configuration and to an analogous extremal point *K*. Moreover, *as a* **new result of this paper**, *an extension onto "absolute plane"* ($\mathbf{S}^2$*,* $\mathbf{E}^2$*,* $\mathbf{H}^2$*,* $\mathbf{M}^2$ *Minkowski plane,* $\mathbf{G}^2$ *Galilei (or isotropic) plane*) *can be formulated and solved by three reflections theorem* (see e.g. Molnár, 1978 and Sect. 4, Weiss, 2018), *and geometric* (Grassmann–Clifford type) *algebra* (Perwass et al., 2004 and Sect. 3). Open problems arise as well. By this we want to follow **F.**

Kárteszi's didactical credo (see also his wonderful book (Kárteszi, 1976) of great international success):

Start with a natural, elementary, visually well understandable task!
Then follow the manipulations, tools, new mathematical concepts,
the technical machinery; then the solution, occasional theory, further applications, extensions ....

Impact of surface continuity on 3D model

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In an engineering environment, continuity is one of the important concepts. Not only does it have a significant impact on the aesthetic aspect of products, which is an essential parameter for commercial success, but it is equally a very crucial parameter from a technological point of view.
The continuity of the geometry of 3D CAD models determines how smooth the transition between two adjacent surfaces of the model is. C2 continuity, i.e. continuity up to the second derivative, has become the standard for the vast majority of CAD software today.
This work focuses on increasing the degree of contiguous surfaces to C3 continuity using CAD software Rhinoceros 7 to improve the quality of the model for CNC processing.

Variations on Fregier´s Theorem

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Fregier´s theorem states: Choose a conic c and a point P on c. The hypothenuses of all right triangles with the right angle at P pass through a single point F, the Fregier point. The set of all Fregier points of a conic is called the Fregier conic. In fact, this is a theorem that belongs to projective geometry. We shall see that there is a close relation to generalized offsets of conics. Futher, we give some generalizations of Fregier´s result.

The family of uniform polyhedra

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Few mathematicians in geometry deal with homogeneous polyhedra. Many of my friends, serious mathematicians find these polyhedrons difficult to create and do not see their use in other fields of knowledge. Hence, little interest in this issue.
However, these polyhedrons are so captivating and beautiful, they require so much hard thought on how to create cardboard models or 3D prints of them.
They were mostly created in the 20th century. They are somewhat analogous to what Archimedean polyhedra are to Platonic polyhedra, with the difference that we now allow faces to be non-convex polygons. There are 54 of them in total, but the last ones were discovered in the 1970s and are very complicated.
In my presentation, on the example of one of the uniform polyhedra, I will show the principle of their construction and I will show the ones whose models I made with my students.

3D printing as a teaching tool in mathematics teaching

Nowadays, many of us know what 3D printing, its design and other technical details are all about. However, it is worth considering how 3D printing can help teach geometry, especially spatial geometry, and even make it easier for students to discover algebraic patterns of abbreviated multiplication.
In my presentation, I will show numerous 3D models, the purpose of which is to better assimilate concepts such as the duality of polyhedrons, stellations, quick determination of the volume of selected solids, the search for geometric relationships between them

and the development of students' spatial imagination by preparing numerous puzzles and 3D puzzles.
All this happens with the participation of a computer that, by designing these numerous spatial structures, teaches the basic geometry of the compass and ruler, because these tools are used when designing these 3D creations in the GeoGebra and SketchUp software.

Konstrukce racionálních čísel pomocí origami

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Potřeba rozdělit papír je snad nejzákladnějším problémem, se který se setkáváme při jakémkoli skládání. Větší nesnáze však přicházejí, má-li být výsledný produkt origami symetrický a vyžaduje tudíž dělení papíru na stejné části. Je nasnadě, že „půlit" papír je velice jednoduché, ale setkáváme se i s problémem, jak zkonstruovat takové dělení, kdy jmenovatel výsledného hledaného zlomku není mocninou čísla 2. V prezentaci ukážeme, jak toho lze dosáhnout pomocí jednoduchých algoritmů.

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Priamkové plochy - tlačené 3D modely vytvorené v prostredí Rhinoceros

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Priamkové plochy sú v stavebníctve a architektúre často využívané pre ich matematickú jednoduchosť a prispôsobivosť. Sú efektívne z hľadiska použiteľného materiálu a pevnosti konštrukcie. Poskytujú možnosť vytvárať jedinečné architektonické prvky, ktoré sú estetické a moderné. Študenti architektúry sú nielen s touto skupinou plôch oboznámení v rámci predmetu Deskriptívna geometria II. Vyučovací proces sme vhodne doplnili skupinou tlačených 3D modelov. V nadväznosti na predchádzajúce skúsenosti pri tvorbe takýchto modelov, naďalej testujeme možnosti ich vytvárania pomocou rôznych vhodných softvérov. V tomto prípade sme modelovanie priamok na týchto plochách reprezentovali pomocou nástroja “pipes“ v prostredí Rhinoceros v kombinácii s nadstavbou Grasshopper.

Selected geometric constructions and proofs

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Geometry has been put on the back burner in many countries, and calculus is used much more. We present some geometric proofs and constructions instead of using calculus. It will be focused on one of the many geometrical proofs of Pappus-Pascal´s theorem and the construction of an osculating circle of an ellipse.

Application of Euler path

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Graph theory is a valuable mathematical modeling tool with a wide variety of geometric connections. It deserves more attention in all types of education. Starting with Euler paths, graph coloring, and Hamilton icosian game pupils can learn the terms and methods important for future programming. This concept can be very useful in real-life applications, such as how to solve transportation problems. In the contribution, the Hamiltonian paths and Eulerian cycles are presented together with their connection to the vehicle routing problems.

On computation of symmetries of algebraic objects

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We will discuss the problem of computation of symmetries of algebraic objects. In many cases the direct approach can be simplified via a reduction to an easier problem. We will demponstate this on the case of symmetries of planar polynomial vector fields.

3D-versions of theorems related to Miquel`s Theorem

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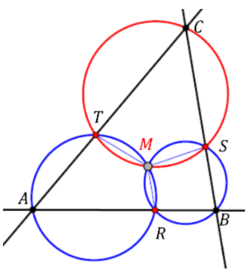
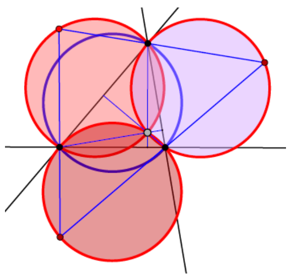
The elementary geometric Miquel theorem concerns a triangle *ABC* and points *R,S,T* on its sides, and it states that the circles ⊙*ART*, ⊙*BRS*, ⊙*CST* have a common point *M*, the Miquel point to these givens. For *M* there exists a two-parametric set of possibilities, such that there exists a one-parameter set of point triplets *R,S.T* to a given point *M*. Choosing *R,S,T* in special ways one receives the so-called *"Beermat theorem"*, the *Brocard theorems*, and the *Simson-Wallace theorem* as special cases of Miquel's theorem. Remaining within Euclidean geometry we deal with 3D modifications of these theorems. It turns out that, while the Miquel theorem can be generalized to n-simplices, 3D versions of the Brocard theorems need some modifications. The 3D Simson-Wallace theorem based on Miquel's theorem is different from the standard generalization (see e.g. P. Pech: Generalization of Simson–Wallace theorem: planar and spatial formulation, JGG 2023), but it connects properties of the 2D case with Brocard's modifications.

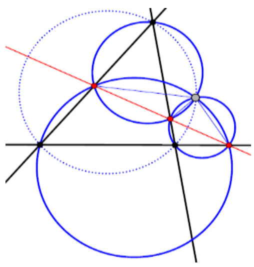
Key words: Brocard points, Miquel's theorem, three-circle-theorem

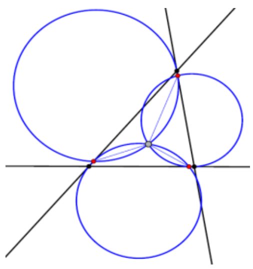
„Miquel-Relatives“



Miquel


Beermat


Steiner, Simson-Wallace


Brocard

Calculating on a parabola

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K. G. Ch. von Staudt described simple geometric constructions of arithmetic operations in his Beiträge zur Geometrie der Lage. We discuss a special case of a parabola in particular. Elementary and derived constructions of addition and multiplication are presented synthetically and analytically, and straightforward algebraic observations are interconnected with the deeper geometric properties of a parabola. We focus on constructions of arithmetic, geometric, and harmonic mean. Von Staudt´s constructions are also discussed in relation to the Matiyasevich-Stechkin parabola and Möbius´ parabolic nomogram.

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Slovenská spoločnosť pre Geometriu a Grafiku

Česká společnost pro geometrii a grafiku

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Richtáriková Daniela – registrácia a administratíva

Szarková Dagmar – webstránka a databáza

Vedecký výbor konferencie

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32. sympózium
o počítačovej geometrii SCG´2023

43. konference
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9. SLOVENSKO - ČESKÁ KONFERENCIA O GEOMETRII A GRAFIKE

Zborník abstraktov

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