Tite of Article

Name Surname1, Name Surname2

Abstrakt
V článku sa zaoberáme .....

Kľúčové slová: kľúčové slová

Abstract
Abstract ...

Keywords: keywords - english

1 Introduction

Text:

In the following, we pay our attention to the special surface that we obtain if we let $a = c = 1$ and $b = d = 0$. This particular surface is the image of the plane $x - z = 0$ under the axial inversion and has the equation

$$x - (x^2 + y^2)z = 0 \tag{1}$$

and was first studied in [**?**].

Obviously, this surface allows two different parametrizations over nearly the same parameter domain $D = \mathbb{R}^\star \times [0, 2\pi[$

2 The minimal surfaces

$$\mathbf{f}(u, v) = \mathfrak{Re}\varphi(t) \tag{2}$$

of the uniquely defined real minimal surface on the scroll (γ, ν).

With the spine curve γ given in (**??**) and the unit normal vector field ν described by (**??**), we can derive the parametrization(s) of the minimal surfaces tangent to MÜLLER's cubic surface. We use shorthand

$$c_x := \cos x, \quad s_x := \sin x, \ldots, C_x := \cosh x, \quad S_x := \sinh x, \ldots$$

together with the abbreviation $p := \sqrt{1 + q^4}$ and state:

Theorem 2.1. The one-parameter family of minimal surfaces touching MÜLLER's surface (**??**) along the ellipses (**??**) can be parametrized over $\mathbb{R}^2$ by

$$\mathbf{f}(u, v) = \frac{1}{6pq} \begin{pmatrix} 6pq^2 c_u C_v - 3(p^2 + q^4) c_u S_v + c_{3u} S_{3v} \\ 6pq^2 s_u C_v - 3(p^2 + q^4) s_u S_v + s_{3u} S_{3v} \\ 6c_u (S_v q^2 + p C_v) \end{pmatrix}. \tag{3}$$

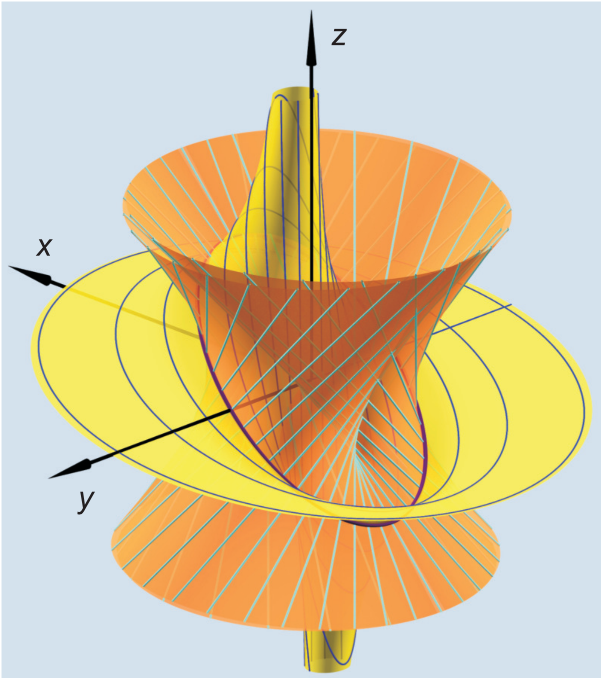

Fig. 1. The sextic ruled surface of normals along an ellipse.

Proof. We insert (**??**) and (**??**) into (**??**) and find the parametrization of the isotropic curve $\varphi:\ \mathbb{R}^3 \to \mathbb{R}^3$

$$\varphi(t) = \frac{1}{6pq}\begin{pmatrix} 6pq^2c_t + 3\mathbf{i}(p^2+q^4)s_t - \mathbf{i}s_{3t} \\ 6pq^2s_t - 3\mathbf{i}(p^2+q^4)c_t + \mathbf{i}c_{3t} \\ 6(pc_t - \mathbf{i}q^2s_t) \end{pmatrix} \tag{4}$$

depending on the complex parameter t. Then, we replace t by $u+\mathbf{i}v$ and extract the real part and obtain (**??**). □

The curves $v=$ const. on the minimal surface have a very special shape:

Theorem 2.2. The u-curves (the curves with $v=$ const.) on the minimal surfaces (**??**) are harmonic oscillation curves and appear as cycloidal curves in the top-view (orthogonal projection onto the $[xy]$-plane).

Proof. We use the first and second coordinate function of $\mathbf{f} = (\mathbf{f}^1, \mathbf{f}^2, \mathbf{f}^3)$ from (**??**) and build the complex variable $w(t) = \mathbf{f}^1 + \mathbf{i}\mathbf{f}^2$ which reads in full length

$$\begin{aligned} w(t) = qc_uC_v - \tfrac{1}{2pq}(p^2+q^4)c_uS_v + \tfrac{1}{6pq}c_{3u}S_{3v} + \\ +\mathbf{i}\left(qs_uC_v - \tfrac{1}{2pq}(p^2+q^4)s_uS_v + \tfrac{1}{6pq}s_{3u}S_{3v}\right). \end{aligned}$$

□

Figure **??** shows the top-views of some of the harmonic oscillation curves on the minimal surfaces given in (**??**).

The following fact is worth to be noted and elementary to verify:

Lemma 2.1. The curves of constant Gaussian curvature on MÜLLER's surface are the ellipses given in (**??**) with $q \in \mathbb{R}^\star$.

Proof. From (**??**) we compute the Gaussian curvature function on MÜLLER's surface and arive at

$$K(q,t) = -\frac{4q^2}{(1+q^4)^2}$$

which is obviously independent of t, and thus, constant along each ellipse for the corresponding fixed $q \in \mathbb{R}^\star$. □

3 The associate family

The parametrizations (**??**) of the minimal surfaces tangent to MÜLLER's surface (**??**) are obtained by extracting the real part of the isotropic curve (**??**). The imaginary part of $\varphi(t)$ (with $t = u + \mathrm{i}v$) is given by

$$\mathbf{f}^\perp(u,v) = \frac{1}{6pq}\begin{pmatrix} -6pq^2 s_u S_v - 3(p^2+q^4)s_u C_v - s_{3u}C_{3v} \\ 6pq^2 c_u S_v - 3(p^2+q^4)c_u C_v + c_{3u}C_{3v} \\ -6s_u(q^2C_v + pS_v) \end{pmatrix}. \tag{5}$$

3.1 The intersection of MÜLLER's surface

The intersection with MÜLLER's surface (**??**) with the minimal surfaces (**??**) along its ellipses contains:

1. the six-fold y-axis (of the underlying Cartesian coordinate system),
2. the two-fold ellipse γ (**??**) (for any fixed $q \in \mathbb{R}^\star$),
3. the three-fold pair of ideal lines of the complex conjugate pair of planes $x^2 + y^2 = 0$, and
4. the eighteen-fold ideal line of all horizontal planes (parallel to $z = 0$).

References

[1] WEISS, G. *Fregier points revisited*, South Bohemia Mathematical Letters 26 (2018), pp. 84–92.

[2] WELLS, D. *The Penguin Dictionary of Curious and Interesting Geometry.* Penguin Science (1992).

[3] LIE, S. *Gesammelte Abhandlungen.* Friedrich Engel, Poul Heegaard, eds.,

[4] MÜLLER, E. Die achsiale Inversion. In *Dt. Math.*, Ver. **25**, 1916, pp. 209–251.

[5] NITSCHE, J.C.C. *Vorlesungen über Minimalflächen.* Springer-Verlag, 1975.

Ing. Name Surname1

Charles University
Ovocný trh 560/5, 116 236 Prague, Czech Republic
e-mail: user1@seznam.cz

prof. Name Surname2, PhD.

University of Bratislava
Nám. slobody 17, 812 31 Bratislava, Slovak RepublicR
e-mail: user2@gmail.com